

# MATHEMATICAL MODELING OF JET VOLCANIC ERUPTION

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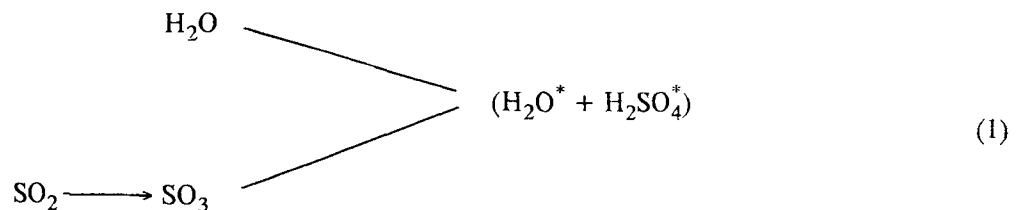
*Results of mathematical modeling of the evolution in the atmosphere of jet-eruption products with the prescribed physical parameters in the outlet section of the volcano crater are reported. Consideration has been given both to low-power eruptions (the most typical ones) and to eruptions of intermediate and high power (catastrophic).*

**Introduction.** Modeling of volcanic eruption is of interest for the prediction of possible consequences (including the formation of atmospheric acid aerosols) and also (by solution of inverse problems) as one approach to the investigation of interterrestrial processes.

We present below the results of mathematical modeling of the evolution in the atmosphere of eruption products with the prescribed physical parameters in the outlet section of the volcano crater. The problem is considered in a two-dimensional formulation (axial symmetry). The physical model considered accounts for nonisothermal unsteady-state motion of air, gaseous eruption products, aerosol, and solid particles in the gravitational field as well as the liquid-vapor transition for water and the dissolution of sulfur oxides with the formation of sulfuric acid. The initial state corresponds to the dry-air atmosphere at rest with a prescribed vertical temperature distribution.

**Model.** It is known that water vapor constitutes a major portion of volcanic gases. Other gases are present in the form of impurities, among which sulfur oxides prevail, as a rule.

With the rise and cooling of the gases, phase and chemical transformations occur in the system: SO<sub>2</sub> oxidation to SO<sub>3</sub> and the formation of a condensate, which is an aqueous solution of sulfuric acid [1]:



It is assumed that the characteristic times of the elementary processes considered are much smaller than the time of rise of a cloud of the eruption products and, accordingly, there are local chemical and phase equilibria in the system. In this case, we can perform a quantitative description with the use of general thermodynamic relations. In particular, the partial pressures of the water vapor  $p_1$  and SO<sub>3</sub>  $p_2$  are related to the sulfuric-acid concentration  $\alpha$  by [2]

$$p_1 = (1 - \alpha) p_{s1}(T), \quad p_2 = \alpha p_{s2}(T), \tag{2}$$

where

$$\alpha = \frac{n_{\text{H}_2\text{SO}_4^*}}{n_{\text{H}_2\text{O}^*} + n_{\text{H}_2\text{SO}_4^*}}; \tag{3}$$

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$p_{s1}$  and  $p_{s2}$  are the pressures of the saturated vapor of pure substances. For  $p_{s1}(t)$  in the calculations we used the following empirical formula [3]:

$$\log p_{s1} = \frac{0.7859 + 0.03477t^c}{1 + 0.00412t^c}, \quad t^c = T - 273.15, \quad (4)$$

in which  $p_{s1}$  is the pressure expressed in millibars. For  $p_{s2}(T)$ , use was made of a dependence similar to (4) but scaled in such a way that the boiling point  $T_{b2}$  of the sulfuric acid corresponds to atmospheric pressure:

$$p_{s2}(T) = p_{s1} \left( T \frac{T_{b1}}{T_{b2}} \right), \quad t_{b1}^c = 100, \quad t_{b2}^c = 296.2 \quad [4]. \quad (5)$$

It is known that the bulk of the condensate exists in the form of fine particles (the maximum of the distribution function falls on sizes of about  $0.1 \mu\text{m}$ ) [5]. Therefore, one need not consider mutual slip of the phases, which can be assumed to move simultaneously. Diffusion is also not considered. In this case, the sulfur-to-hydrogen ratio is retained:

$$\left( \sum_i v_i^2 n_i \right) / \left( \sum_i v_i^1 n_i \right) = \xi. \quad (6)$$

The majority of the volcanos investigated are characterized by  $\xi \approx 0.03$  [5], although in the recent eruption of Popocatepetl (Mexico) an abnormally high concentration of  $\text{SO}_2$  was noted [6].

The pressures and densities are related by the gas equations

$$p_1 = n_1 RT, \quad p_2 = n_2 RT, \quad p_a = \frac{\rho_a}{\mu_a} RT. \quad (7)$$

The total density of the eruption products obeys the continuity equation

$$\frac{\partial \rho_p}{\partial t} + \text{div}(\rho_p \mathbf{v}) = 0, \quad \rho_p = \sum_i \rho_i, \quad \rho_i = \mu_i n_i. \quad (8)$$

A similar equation describes the dynamics of the air density

$$\frac{\partial \rho_a}{\partial t} + \text{div}(\rho_a \mathbf{v}) = 0. \quad (9)$$

The components of the velocity vector satisfy the equations of motion

$$\frac{\partial}{\partial t}(\rho v_r) + \text{div}(\rho v_r \mathbf{v}) = -\frac{\partial p}{\partial r}, \quad \frac{\partial}{\partial t}(\rho v_z) + \text{div}(\rho v_z \mathbf{v}) = -\frac{\partial p}{\partial z} - \rho g, \quad (10)$$

where

$$\rho = \rho_a + \rho_p, \quad p = p_a + p_1 + p_2. \quad (11)$$

Finally, the mixture temperature is determined from the energy equation

$$\frac{\partial}{\partial t} \left[ \rho \left( \frac{v^2}{2} + e \right) \right] + \text{div} \left[ \rho \left( \frac{v^2}{2} + e \right) \mathbf{v} \right] = -\text{div}(p\mathbf{v}) - \rho g v_z \quad (12)$$

and the calorific equation of state

$$e(T) = \sum_i (\kappa_i + c_i T) \rho_i. \quad (13)$$

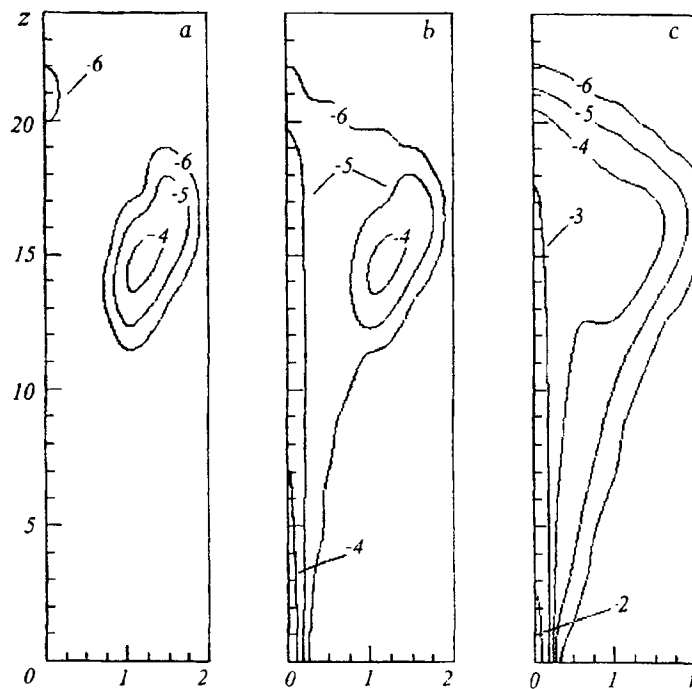


Fig. 1. Isolines of the logarithm of condensate density at an instant of 5 min after the beginning of eruption vs. sulfur oxide content in the eruption products: a)  $\text{SO}_2/\text{H}_2\text{O} = 0$ ; b) 0.001; c) 0.03.  $r, z$ , km.

Relations (2)-(13) form a closed system of equations for the parameters of state considered as functions of time  $t$  and the  $r$ - and  $z$ -coordinates.

The initial state corresponds to the dry-air atmosphere at rest with the standard distribution of the parameters throughout the height [7]:

$$t = 0: \mathbf{v} = 0, T = T_e(z), \rho_p = 0, \rho_a = \rho_e(z). \quad (14)$$

The boundary conditions are as follows:

$$z = 0, r \leq r_c: v_z = v_c, v_r = 0, T = T_c, p = p_c, \rho_a = 0 \quad (15)$$

(for the volcano crater);

$$z = 0, r > r_c: v_z = 0 \quad (16)$$

(for the earth's surface)

$$z = z_{\max} \text{ or } r = r_{\max}: T = T_e(z), \rho_p = 0, \rho_a = \rho_e(z) \quad (17)$$

(the lateral and upper boundaries of the calculated region).

With the use of the above model we performed calculations for a cylindrical region of space. The system of equations (2)-(13) with initial and boundary conditions (14)-(17) was solved numerically on uniform difference grids. A discrete analog of the differential equations was constructed in accordance with Patankar's concept [8], which implies the employment of the laws of conservation in integral form for physical quantities averaged over the volumes of elementary cells; the mean scalar parameters of state are related to the centers of the cells while the vector components – to the centers of the corresponding faces. In calculating the flows across the faces, use was made of the "counterflow" scheme. Time integration was accomplished by a second-order Runge-Kutta method.

**Calculation Results. Low-power eruptions.** A calculation of low-power eruption has been carried out for the following parameters in the outlet section of the volcano crater: outflow velocity 10 m/sec, temperature 1400 K [5], pressure 1 atm, diameter of volcano crater 250 m, which corresponds to an intensity of mass ejection of 80 tons/sec

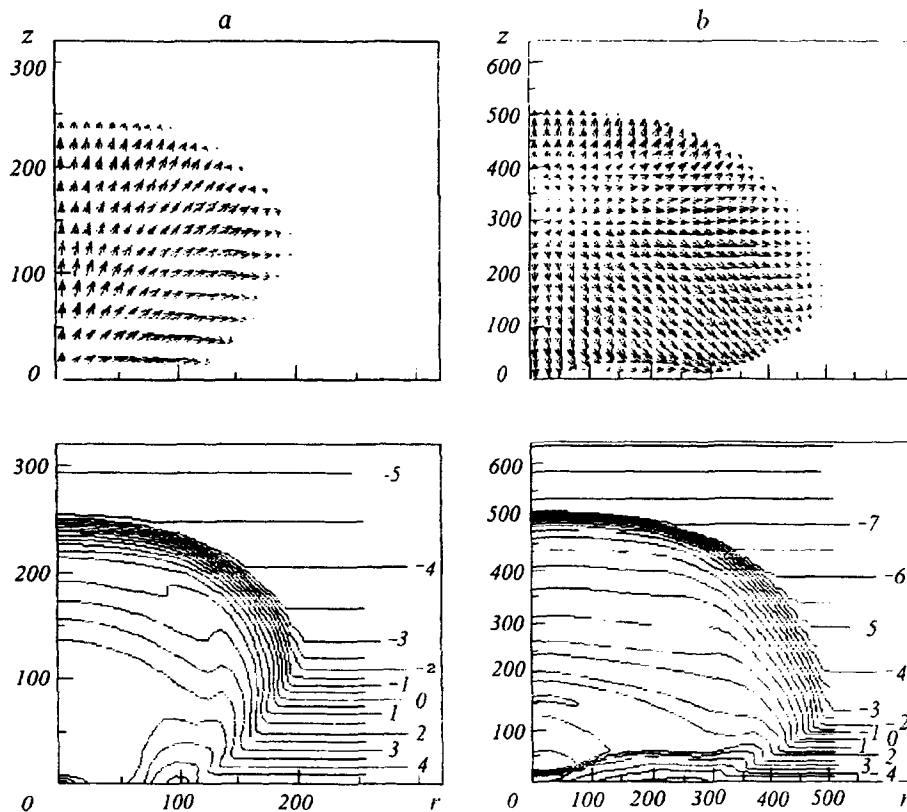


Fig. 2. Velocity fields and isolines of the logarithm of pressures at times of 2 min (a) and 5 min (b) after the beginning of eruption.  $p$ , Pa.

and a power of 120 GW (here, the densities of the mass and energy fluxes correspond, for instance, to the eruption of the Karymskii volcano (Kamchatka, 1966) [9].

When the  $\text{SO}_2$  concentration in the eruption products is low ( $\xi \ll 1$ ) the specific weight of the volcanic gases is smaller than that of air even after their cooling, which is responsible for the increase in the speed of rise of the cloud of eruption products in its initial stage due to the buoyancy force. As the height of rise increases, the influence of the buoyancy force decreases due to the cooling of the volcanic gases and mixing with air. However, at the same time the decrease in the air density with height begins to exert an influence, which leads to a further increase in the gas velocity.

The character of condensate formation (Fig. 1) depends significantly on the  $\text{SO}_2/\text{H}_2\text{O}$  ratio in eruption products. When  $\text{SO}_2/\text{H}_2\text{O} = 0$ , condensation is absent in the initial stage, and ( $t = 3$  min,  $z = 10$  km) an abrupt increase in the intensity of condensate formation occurs. The condensation region acquires the form of a dome, which after a time breaks down into a cloud, which continues to rise, and into a torus, which expands in the radial direction. In both regions, the amount of the condensate per unit volume gradually decreases.

With  $\xi = 0.03$  (Fig. 1c) condensation occurs even in the immediate vicinity of the crater. The condensate is a concentrated solution of sulfuric acid ( $\alpha \approx 80\text{--}90\%$ ). Practically all the sulfur passes into solution. The condensation region has the form of an upward-directed flame, and the content of the condensate per unit volume decreases monotonically with height.

For the intermediate value  $\xi = 0.001$  (Fig. 1b), there is a superposition of the characteristic features inherent in both cases considered above: with  $t \approx 4$  min at a height of approximately 4 km a region of more pronounced water condensation develops against the background of the flame of sulfuric acid aerosol similar to that for  $\xi = 0.03$ , which leads to a stepwise increase in the total condensate mass and to a substantial decrease in the acid concentration ( $\alpha < 0.5\%$ ) in this region.

*Intermediate-power eruptions.* A series of calculations of jet eruption of a gaseous volcano has been carried out for different values of the initial parameters corresponding to a power ( $A$ ) of  $1.2 \cdot 10^{12}$  W (crater diameter 250 m, initial outflow velocity 100 m/sec, initial temperature of eruption products 1400 K, pressure 1 atm;  $\text{SO}_2:\text{H}_2\text{O} =$

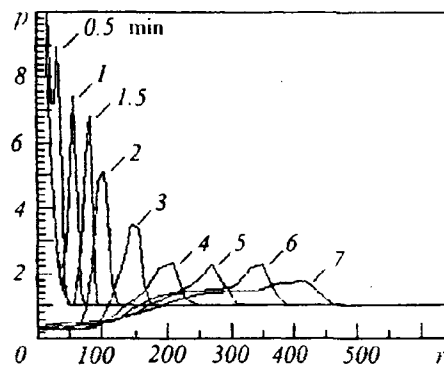


Fig. 3. Pressure distribution on the earth's surface for different times.  $p$ , atm.

0, 0.001, 0.03) and (B)  $1.2 \cdot 10^{17}$  W (crater diameter 2.5 km, initial outflow velocity 1 km/sec, initial temperature of eruption products 1400 K, pressure 100 atm).

Comparative characteristics of the process for an instant of 1 min after the beginning of the eruption are as follows:

- energy release  $72 \cdot 10^{12}$  and  $72 \cdot 10^{17}$  J;
- displacement of the atmospheric-disturbance front 9 and 60 km in the vertical direction and 2 and 35 km in the horizontal direction;
- maximum gas velocity 260 m/sec at a height of 4 km and 2.5 km/sec at a height of 30 km;
- height of rise of the cloud of eruption products 8 and 56 km;
- radius of the cloud of eruption products 0.6 and 35 km.

For case A, the form of the condensate clouds as a function of the  $\text{SO}_2\text{:H}_2\text{O}$  ratio is similar to that in a volcanic eruption of lower power considered earlier. For case B, the condensate region is an ellipsoidal layer from 20 to 40 km in height and from 17 to 26 km along the horizontal. The condensate density in the layer is up to  $10 \text{ g/m}^3$ . The pressure jump in the shock wave at the earth's surface amounts to approximately 15% in relation to the atmosphere at rest.

*High-power eruptions.* Results provided in Figs. 2 and 3 correspond to volcanic eruption with a total energy release of  $3.2 \cdot 10^{21}$  J (crater diameter 10 km, outflow velocity 1 km/sec, crater pressure  $10^8$  Pa, temperature 1400 K, eruption time 97 sec), the eruption products of which (total mass  $1.25 \cdot 10^{15}$  kg) consist of water vapor or water vapor and solid particles (10% in mass) for different sizes of particles.

The dynamics of the solid phase was described within the framework of the multiveLOCITY-continuum model. The above model of gas motion is modified as follows: additional parameters are introduced, which characterize the state of the disperse phase (the mass density, the velocity, and the density of the internal energy, respectively); additional equations, which express the laws of conservation of mass, momentum, and energy of the disperse phase, are introduced; the terms describing the force interaction and heat transfer between the gas and the particles [10] are added to the equations for momentum and energy of the gas.

The gasdynamic picture of eruption and the dynamics of condensate formation in both cases (of purely gaseous eruption and eruption with particles) are close.

The quantity of settled out particles at an instant of 7 min after the beginning of the eruption amounts to 0.15% of their total mass if the particle diameter is 0.1 mm. With a size of the fragments of about 1 m at the same instant, 99.9% of their mass precipitate, the maximum of the precipitation intensity is reached at  $t = 5.5$  m; and pronounced precipitation is observed in the zone with a radius of 250 km.

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## NOTATION

$t$ , time;  $r$  and  $z$ , cylindrical coordinates;  $\xi$ , initial molar ratio  $\text{SO}_2\text{:H}_2\text{O}$  in volcanic gases;  $\nu_i^1$  and  $\nu_i^2$ , stoichiometric coefficients for the  $i$ th substance, respectively, for water and  $\text{SO}_2$ ;  $\alpha$ , sulfuric-acid concentration;  $n_i$ ,

number of moles of the substance per unit volume;  $\rho$ , density;  $v$ , velocity;  $T$ , absolute temperature;  $t^c$ , temperature, °C;  $T_{b1}$  and  $T_{b2}$ , boiling points of water and sulfuric acid, respectively;  $p$ , partial pressures of gaseous substances;  $p_s$ , pressure of saturated vapors of pure substances;  $e$ , internal energy;  $\mu$ , molecular mass;  $c$ , specific heat capacity;  $\kappa_i$ , specific energy of formation of the  $i$ th substance;  $g$ , free-fall acceleration;  $R$ , universal gas constant. Superscript and subscripts:  $i$ , substance number; p, eruption products; a, air; s, saturation; b, boiling; \*, condensed substances; c, volcano crater; e, equilibrium.

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